Aggregation with a non-convex labor supply decision and habits in consumption

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Abstract

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy where households feature consumption habits. We show how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and aggregate over individual preferences. The presence of habits in consumption does not affect the results. As in Hansen (1985) and Rogerson (1988) and no consumption habits, with a discrete labor supply decision at individual level, the elasticity of hours worked at the aggregate level increases from unity to infinity.

Keywords: Aggregation, Indivisible labour, Consumption habits, Non-convexities

JEL Classification Codes: E1, J22, J46

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1 Introduction and Motivation

The purpose of this paper is to explore the problem of non-convex labor supply decision in an economy where households feature consumption habits. We show how lotteries as in Rogerson (1988) can again be used to convexify consumption sets, and aggregate over individual preferences. The presence of habits in consumption does not affect the results. As in Hansen (1985) and Rogerson (1988) and no consumption habits, with a discrete labor supply decision at individual level, the elasticity of hours worked at the aggregate level increases from unity to infinity.

2 Model setup

The economy is static, there is no physical capital, and agents face a non-convex labor supply decision. Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by i and distributed uniformly on the $[0, 1]$ interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index i to save on notation.

2.1 Description of the model

Each household maximizes the following utility function:

$$U(c, l) = \ln(c - \phi c_{-1}) + \alpha \ln l, \tag{1}$$

where c denotes current consumption of market output, c_{-1} is past consumption (taken as given), $0 < \phi < 1$ denotes the persistence in consumption, or the strength of the consumption habits; l is the leisure enjoyed by each individual household, and $\alpha > 0$ is the relative weight attached to utility of leisure. Each household is endowed with a time endowment of unity, which can be split between hous worked, h, and leisure l, so that

$$h + l = 1 \tag{2}$$

The households make a discrete labor supply choice: whether to work full-time, or not at all. In other words, $h \in \{0; \bar{h}\}$, where $0 < \bar{h} < 1$. The hourly wage rate is denoted by w.

Finally, the households own the representative firm, and are entitled an equal share of the profit (π).

The problem faced by a household that decides to work full-time in the market sector is then to set $h = \bar{h}$ and enjoy

$$U^w = \ln(w\bar{h} + \pi - \phi c_{-1}) + \alpha \ln(1 - \bar{h}), \tag{3}$$

while a household that decides not to work obtains

$$U^u = \ln(\pi - \phi c_{-1}) + \alpha \ln(1) = \ln(\pi - \phi c_{-1}) \tag{4}$$

2.2 Stand-in firm: market sector

There is a representative firm in the model economy, which operates in the market sector. It produces a homogeneous final product using a production function that requires labor H as the only input. For simplicity, output price will be normalized to unity. The production function $f(H)$ features decreasing returns to scale: $f'(H) > 0$, $f''(H) < 0$, $f'(0) = \infty$, $f'(\bar{h}) = 0$. The representative firm acts competitively by taking the wage rate w as given and chooses H to maximize profit:

$$\pi = f(H) - wH \quad \text{s.t.} \quad 0 \leq H \leq \bar{h}. \tag{5}$$

In equilibrium, there will be positive profit, which follows from the assumptions imposed on the production function.

2.3 Decentralized competitive equilibrium (DCE): Definition

A DCE is defined by allocations $\{c^w, c^u, c_{-1}, h\}$, wage rate $\{w\}$, and aggregate profit ($\Pi = \pi$) s.t. (1) all households maximize utility; (2) the stand-in firm maximizes profit; (3) all markets clear.

3 Characterization the DCE and derivation of the aggregate utility function

It will be shown that in the DCE, if it exists, only some of the households will be employed and work full-time, while the rest will be unemployed. Following the arguments in Rogerson (1988) and Hansen (1985), it can be easily shown that polar cases in which each household either works in the official, or in the unofficial sector, cannot not be equilibrium outcomes. Therefore, it must be the case that a proportion of the agents in the economy are working, while the rest are not. Denote this mass of officially employed by λ, and the officially unemployed by $1 - \lambda$. Workers in the official sector will receive consumption c^w, while those unemployed will consume c^u. Note that λ can be interpreted also as the probability of being chosen to work: This probability is determined endogenously in the model, as workers would seek for the optimal balance between the net return from working and lesiure (at the margin). No matter of the employment outcome, ex post every household enjoys the same utility level.

Thus, in equilibrium $H = \lambda\bar{h}$. From the firm's point of view then the wage is set equal to:

$$w = f'(\lambda\bar{h}) \tag{6}$$

Firm's profit is then

$$\pi = \Pi = f(\lambda\bar{h}) - f'(\lambda\bar{h})\lambda\bar{h} > 0, \tag{7}$$

which follows from the decreasing returns to scale featured by the production function. Next, to show that the DCE actually exists, it is sufficient to show the existence of a fixed point $\lambda \in (0, 1)$ by analysing a non-linear equation using the fact that in equilibrium utility is the same for all households. In particular, it is trivial to show that everyone working in the market sector ($\lambda = 1$) is not an equilibrium, since then $w = 0$. From the ex ante symmetry assumption for households, market consumption would be the same for both market workers and those not selected for work, while the latter would enjoy higher utility out of leisure (holding h fixed), hence there is no benefit of working. Similarly, nobody working in the market sector ($\lambda = 0$) is not an equilibrium outcome either, since the firm would then offer a very high wage for the first unit of labor, and by taking a full-time job a marginal worker could increase his/her utility a lot.

Thus, if there is a DCE, then it must be that not all households would receive the same consumption bundle. If $\lambda \in (0,1)$ is an equilibrium, then total utility for households that work in the market sector should equal to the utility of households that do not supply any hours in the market sector. This equation is monotone in λ, as the utility function is a sum of monotone functions. Thus we can explore the behavior of that function as we let λ vary in the $(0,1)$ interval. As $\lambda \to 0$, the left-hand-side dominates (utility of working is higher), while when $\lambda \to 1$ the right-hand-side dominates (utility of not working is higher), where the results follow from the concavity of the utility functions and the production technologies. In addition, from the continuity of those functions, $\exists \lambda \in (0,1)$, which is consistent with equilibrium. The unique value of λ follows from the monotonicuty of the utility and production functions. Let c^{m*} and c^{u*} denote equilibrium consumption allocations of individuals selected for work, and those who will not work.

Given the indivisibility of the labor supply in the market sector, the equilibrium allocation obtained above is not Pareto optimal, as demonstrated in Rogerson (1988). More specifically, a social planner (SP) could make everyone better off by using an employment lottery in the first stage and choosing the fraction λ of individual households to work in the market sector and give everyone consumption $\lambda c^{m*} + (1-\lambda)c^{u*}$. In order to show this, we need to check that such an allocation is feasible, and that it provides a higher level of total utility. Showing feasibility is trivial as total market labor input and total consumption are identical to the corresponding individual equilibrium values.

Next, we will show that the new allocation - which is independent of a household's employment status - makes households better off since it generates higher utility on average. This is indeed the case, where the strict inequality follows from the convexity of the production function and the concavity of the logarithmic function. Thus, the SP is indeed giving in expected utility terms an allocation that is an improvement over the initial equilibrium allocation. If households can pool income together and doing so, they will be able to equalize consumption across states, i.e., $c = c^{m*} = c^{u*}$:

$$\ln(c - \phi c_{-1}) + \lambda\alpha \ln(1 - \bar{h}). \tag{8}$$

Observing that for the aggregate household $H = \lambda\bar{h}$, substituting the expression in the aggregate utility above and rearranging terms yields

$$\ln(c - \phi c_{-1}) + AH, \tag{9}$$

where

$$A = \frac{\alpha \ln(1 - \bar{h})}{\bar{h}} < 0 \quad as \quad \ln(1 - \bar{h}) < 0 \tag{10}$$

The resulting aggregate utility function is of an interesting and novel form. On the aggregate, when each household faces an indivisible labor choices, the representative agent obtained from the aggregation features different preferences of work in terms of aggregate hours. Interestingly, the result is not affected by the presence of consumption habits.

4 Conclusions

This paper explored the problem of discrete labor supply decision in an economy with consumption habits. We show how employment lotteries as in Rogerson (1988) can still be used to convexify consumption sets, and aggregate over individual preferences. The presence of habits in consumption does not affect the results. As in Hansen (1985) and Rogerson (1988) and no consumption habits, with a non-convex labor supply decision at individual level, the elasticity of hours worked at the aggregate level increases from unity to infinity.

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